Management of Uncertainties at the Level of Global Design

Erick Herbin, Julien Jakubowski,
Michel Ravachol and Quang Dinh
Dassault Aviation
Direction de la Prospective
78 quai Marcel Dassault
92552 Saint-Cloud Cedex 300
France

erick.herbin@dassault-aviation.fr, julien.jakubowski@dassault-aviation.fr,

michel.ravachol@dassault-aviation.fr and quang.dinh@dassault-aviation.fr

ABSTRACT

At the conceptual stage of the design of an aircraft, the prescribed performances suffer for uncertainties due to the non-knowledge of the whole geometric parameters and approximations in models involved. In this paper, we present a design process that allows controlling the risk of non-validity of specified performances. In this frame, we follow the methodology commonly used in industrial context. We show that the choice of the uncertainty propagation method influences deeply the designed aircraft that will satisfy the prescribed performances with some high probability.

We also initiate a methodology to use CFD results to improve the uncertainty assessment and then to improve the robust design process of the aircraft.

1.0 PRELIMINARY DESIGN OF A BUSINESS JET

At the early stage of the design of a business jet, the choice of global parameters aims at satisfying different performance targets. The design process relies on interactions between several physical models that represent phenomena involved in the performances: e.g. aerodynamics, structural mechanics and engine performance.

At Dassault-Aviation, the conceptual design uses an integrated computing platform, which performs the optimization of selected performance targets under the constraint of global specifications. In this paper, we focus on the choice of the fuel quantity (driven by the fuselage length, see figure 1) to insure a range not less than 4000 NM.

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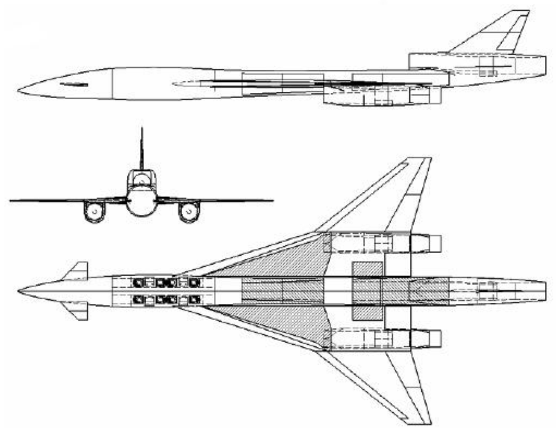

Figure 1: Conceptual sketch of a supersonic aircraft

In this case, the design process is described in figure 2. The design loop takes a parameter *Range Design* as an input, which represents the given range target, and computes the global parameters that define the aircraft.

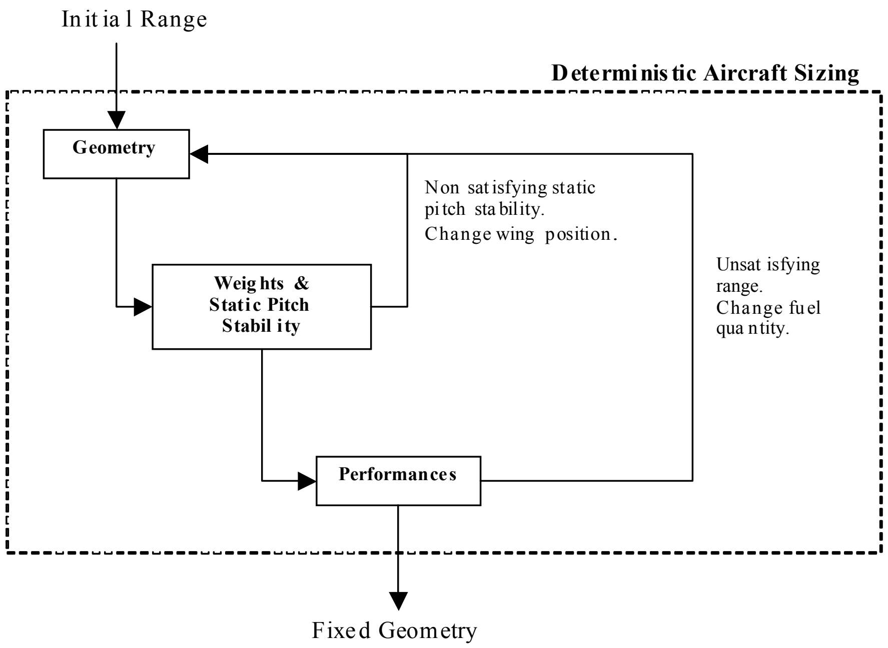

Figure 2: Conceptual design process of an aircraft

At the conceptual design stage, the predicted performances suffer from a certain level of uncertainty, which can be related to two major sources:

- The entire set of sizing parameters is not fully determined yet,
- At this stage, the computational models can be simplified and therefore can only approximate the actual physical behaviour.

Consequently, the effective range of the aircraft can differ from the *Range Design*. There is a possibility that the designed aircraft does not satisfy the specified performance. For a long time, conceptual designers used to add *fixed margin* to global parameters to cover this risk.

To better manage the risks, both technical and economical, one can benefit from using a probabilistic framework at the earliest stage of a program. The first step was to replace the usual 'fixed margin' approach by probabilistic criteria, which enables to relate the margins to a good description of uncertainties involved in the design process.

This is a required step towards a robust optimization, which is simply the optimization of global parameters for a not fully known situation.

2.0 CONCEPTUAL FRAMEWORK OF THE UNCERTAINTY STUDY

The probabilistic framework commonly used in various industrial fields for uncertainty management is appropriate to provide robustness to performances prescribed at the conceptual design stage. We refer to [2] for a comprehensive description of the methodology.

According to [2], the robustness *final goal* of the uncertainty assessment must be seek through the following steps:

- Choice of the *variables of interest*: the global variables, here the range of the aircraft, that determine the specified performance.
- Choice of the *quantity of interest* and *decision criteria*: the measure of uncertainty on the variables of interest, e. g. the probability to exceed a safety/performance threshold, confidence interval or variance.
- Definition of the deterministic *pre-existing model*: the physical or industrial system which computes the variables of interest from uncertain variables.
- *Uncertainty modelling*: choice of the formalism to describe uncertainties, list the sources of uncertainty and quantify them.
- *Propagation of uncertainties*: quantification of uncertainties on the output variables of the pre-existing model, due to sources of uncertainty.
- *Sensitivity analysis*: classification of the sources of uncertainty with respect to their impact on quantity of interest.

In the following paragraphs, we detail the previous step in our case of global design.

2.1 System modelling

From the designed aircraft resulting from the process described in paragraph 1.0, we have to compute its range. At the global design stage, we use a simplified model relying on Breguet range equation (which gives range of an aircraft in climbing cruise),

$$R = \frac{V}{gC_s}\frac{C_L}{C_D}\log\left(\frac{TOW}{ZFW}\right).$$

In this expression, the variables involved are:

- $\boldsymbol{V}$: Cruise velocity [m/s];
- $\boldsymbol{g}$: Standard gravity [m/s^2];
- $\boldsymbol{C_s}$: Specific fuel consumption [kg/(N.s)];
- $\boldsymbol{C_L}$: Lift coefficient ;
- $\boldsymbol{C_D}$: Drag coefficient computed by $C_D = C_{balance}\left(C_{D\min} + KC_L^2\right)$, where
  - $\boldsymbol{C_{balance}}$ is a multiplicative factor on $\boldsymbol{C_D}$ which represents the effect of the load dispatch;
  - $\boldsymbol{C_{D\,\min}}$ is the minimum drag coefficient;
  - $\boldsymbol{K}$ is the drag-due-to-lift factor or polar factor;
- $\boldsymbol{TOW}$: Take-off weight [kg];
- $\boldsymbol{ZFW}$: Zero fuel weight [kg].

A flight is usually composed of several phases, e.g. take-off, climb, cruise, descent, ... Here we only consider the climbing cruise, but the methodology described in this paper would be similar with a more detailed performance model.

Let us remark that several models are involved in the design process to get an aircraft which range is not less than 4000 NM. The uncertainty assessment considers that the design aircraft resulting from the design loop described in paragraph 1.0 is fixed, and we study uncertainties that occur in range computation.

2.2 Uncertainty modelling

We use the probabilistic framework to describe uncertainties involved in the design process. Then, to describe the fact that a quantity is uncertain, we consider it as a random variable.

We now have to list the various sources of uncertainty, and to quantify it by the probability distributions of the random variables.

2.2.1 Sources of uncertainty

Some variables in Breguet range equation are uncertain at the preliminary stage of design. Among these, we focus on: $\boldsymbol{C_s}$, $\boldsymbol{C_{balance}}$, $\boldsymbol{C_{D\,\min}}$, $\boldsymbol{K}$ and $\boldsymbol{ZFW}$.

2.2.2 Quantification of the sources of uncertainty

In the context of global design, it is often difficult to obtain enough data for estimating the probability distributions of the random variables. Then we have to take into account any information or expert judgment about uncertainties.

However some random variables can be computed from a sharper level of modelling. For instance, in the design process, the drag coefficient is computed by CFD software. This point is detailed in paragraph 4.0.

Here, in absence of large amount of data, we consider that the values computed by the design process (paragraph 1.0) determine the expectation of random variables. Moreover, aerodynamics and preliminary design specialists provide intervals on these uncertain variables. Considerations of Information Theory entropy, such as entropy maximization, suggest describing the uncertainty with a triangle probability distribution. However, for regularity purpose, we choose a normal (i.e. Gaussian) distribution for the five uncertain variables involved in Breguet's performance model.

2.3 Decision criteria, variable and quantity of interest

The choice of the variables of interest comes directly from global specifications. Here we focus on the specified performance on the range, which must be not less than 4000 NM. The quantity of interest represents the measure of uncertainty and is the mathematical expression of the risk that we want to cover. The relevant quantity is then the probability that the range is lower than 4000 NM,

$$Q = P(R < 4000 \text{ NM}).$$

Using the safety vocabulary, the subset $F = \{x \in \mathbf{R}^n : R(x) < 4000NM\}$ of $\mathbf{R}^n$ is called the failure space and our purpose in the context of aircraft design is to build a design process such that the probability of the event $\{X \in F\}$ is as small as possible.

For the sake of graphical illustration let us consider the simplified problem with two uncertain variables: the minimum drag coefficient and the zero fuel weight. In this case, the risk of not satisfying the specification $R(X) > 4000NM$ (range below a target) is measured by the probability $P\{X \in F\}$ where $F = \{x \in \mathbf{R}^2 : R(x) < 4000NM\}$. This probability can be computed by a Monte-Carlo method, requiring a suitably large number of random samples of X (figure 3).

The target value of the quantity of interest represents a measure of the risk that the industrial accepts for the non-validity of some specified performance. Although the Program Manager can only make this choice, we choose a value of 0.95, for sake of methodology illustration, as the aimed probability that the range is lower than 4000 NM.

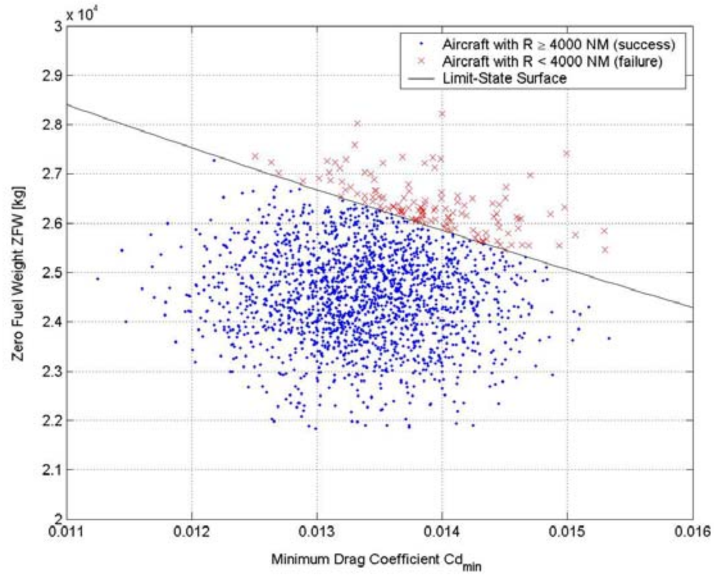

Figure 3 : Sampling representation of Min. Drag Coef. and Zero Fuel Weight with respect to failure space R<4000 NM.

2.4 Propagation and sensitivity analysis

The uncertainty propagation is the step which determines a measure of the predicted performances uncertainty. For that purpose, the global design variables are supposed fixed and the uncertainties are studied through the Breguet's performance model (figure 4).

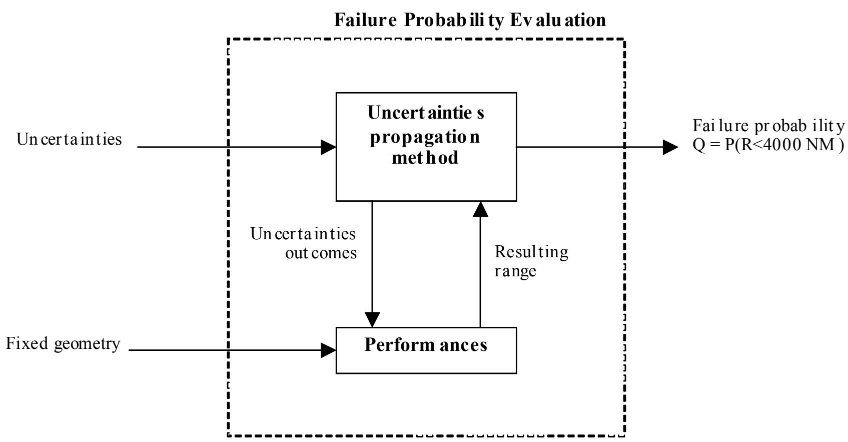

Figure 4: Uncertainty propagation in the design process

Uncertainty propagation methods have been developed and implemented in several industrial conceptual design platforms. Following the well known Monte-Carlo method, approximated methods derived from

Taylor expansion or from structural reliability analysis have been implemented. The FORM/SORM (First and Second Order Reliability) methods rely on a local approximation of the limit state surface by an hyper plane (First Order Method) or a conic (Second Order Method).

The complexity of models involved in preliminary design allows the use of the three propagation methods. We will see in paragraph 2.0 that the three propagation methods induce three different designs of the aircraft.

2.4.1 Monte-Carlo method

Monte-Carlo method is the more natural and well-known method of uncertainty propagation. It consists of a sampling $(X_1, X_2, ..., X_n)$ of the random variable X that represents the uncertain inputs of the model, and the computation of $(R(X_1), R(X_2), ..., R(X_n))$, that forms a sampling of the random variable $R=R(X)$.

This sampling allows approximate the probability distribution of R by the empirical distribution (figure 5). Therefore, the quantity of interest $P(R < 4000 \text{ NM})$ can be approximated by

$$P(R < 4000 \text{ NM}) \approx \frac{\#\{\mathrm{i} : \mathrm{R}(\mathrm{x_i}) < 4000NM\}}{n} = \sum_{i=1}^{n} \mathbf{1}_{\{R(x_i) < 4000NM\}}.$$

Since the precision of the previous approximation increases with the size of the sampling, Central Limit Theorem allows to link the error to n (determination of a 95% confidence interval of relative precision equal to 5%, see [4]).

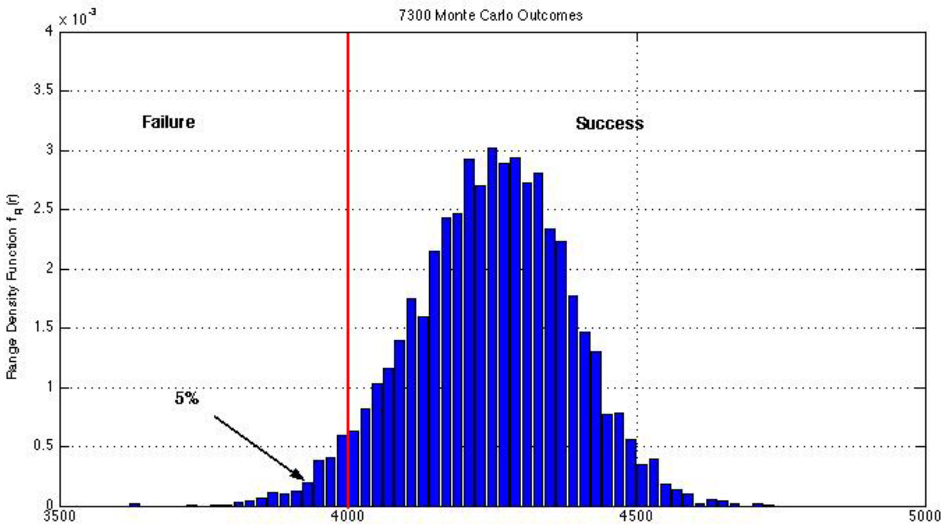

Figure 5: Empirical distribution of the range, from Monte-Carlo sampling.

Monte-Carlo method provides a classification of the sources of uncertainty in function of their influence on range uncertainty. More precisely, we compute the correlation coefficient between each uncertain input variable and the range (table 1).

X_i	Corr(X_i,R)
ZFW	-0.66
$C_{D\,min}$	-0.61
C_s	-0.32
K	-0.20
$C_{balance}$	-0.07

Table 1: Classification of sources of uncertainty.

This classification gives the variables whose uncertainty is the most influent on the range uncertainty. Then, the risk of non-validity of specified performance could be reduced either with a finest knowledge of the most influent variables or in changing the design such that the most uncertain variables are less influent.

2.4.2 Taylor quadratic approximation method

When the pre-existing model is too CPU costly, e.g. when it relies on a finite element code, the Monte-Carlo method cannot be used to propagate uncertainties. However, some approximated methods can be used. One of them consists in a first order Taylor expansion of the variable of interest, around the mean value of the uncertain variables ***X***, (see [3])

$$R(X) = R(X^0) + \sum_{i=1}^{N} \left(\frac{\partial R}{\partial X_i}\right)_{X^0} (X_i - X_i^0) + o\left(\left\| X - X^0 \right\|\right)$$

The random variable R(X) is then described by its two first moments

- $E[R(X)] = R(X^0)$, and
- $Var[R(X)] = \sum_{i,j} \left(\frac{\partial R}{\partial X_i}\right)_{X^0} \left(\frac{\partial R}{\partial X_j}\right)_{X^0} Cov(X_i, X_j)$,

  that becomes $Var[R(X)] = \sum_{i=1}^{N} \left(\frac{\partial R}{\partial X_i}\right)_{X^0}^2 \sigma_{X_i}^2$ if the $\boldsymbol{X_i}$ are independant.

In order to estimate the probability $P(R < 4000\ \text{NM})$ from the only two first moments of ***R(X)***, we need to assume that the probability distribution of ***R(X)*** is Gaussian (recall that Gaussian variables are the only random variables whose distribution is completely characterized by its two first moments) . Under this assumption, we can estimate a confidence interval or a probability to exceed a threshold.

Since a Monte-Carlo method has already been applied in our case, we can verify the assumption of Gaussian distribution of ***R***. We use a statistical test to compare the empirical distribution from the Monte-Carlo sampling obtained in paragraph 2.4.1 to a given Gaussian distribution. The conclusions of the χ^2 test say that the range of the aircraft does not fit a Gaussian distribution. Consequently, the Taylor quadratic approximation method should lead to a different set of values for design variables than those computed from Monte-Carlo method (paragraph 3.0).

2.4.3 FORM/SORM methods

When computational costs prohibit such a large sampling, another approximated methods, such as the FORM/SORM methods, allow us to estimate the desired probability (see [1, 5]). In this case, after transforming the random variables X into normalized variables U, the probability is computed using the reliability index β, which is the distance of the origin in the U space to the boundary of the limit state $\partial F = \{ \mathrm{x} \in \mathbf{R}^{\mathrm{n}} : \mathrm{R(x)} = 4000\mathrm{NM} \}$.

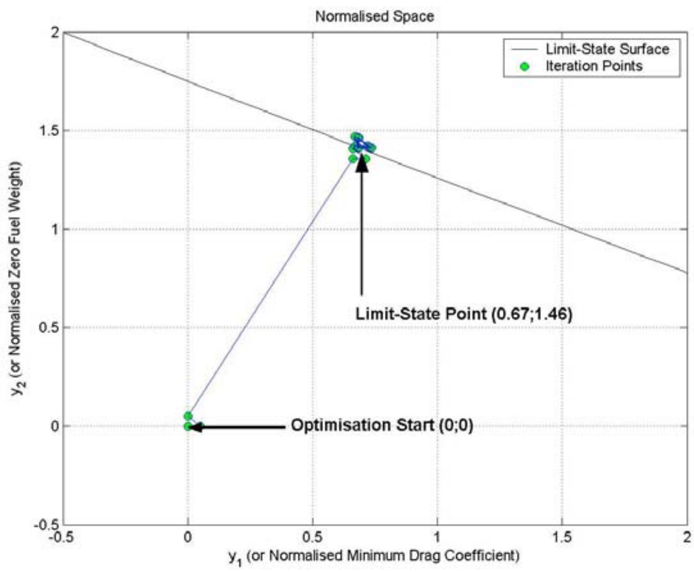

Figure 6: Constrained optimization to estimate the probability of failure (FORM method).

In practical applications, the boundary of the limit state is not a known function and needs to be approximated. The FORM method approximates this limit state surface with an hyperplane, whereas the SORM method use a conic surface approximation. Among the different method to converge to the limit point, a Kriging model can be used to approximate the physical model and to compute efficiently the reliability index β.

The mathematical problem reduces then to a constrained minimization problem. In that view, the iterative process searches the minimum distance between the failure space and the origin (figures 6 and 7). This leads to a costless process than Monte-Carlo, where the iterations consist in an integral computation. In this case, the solution is obtained in 16 iterations of the global sizing model.

In this example, we have also compared the FORM/SORM and Monte-Carlo methods, using 7300 samples to estimate the probability for Monte-Carlo (each evaluation of a sampling data requires the solution of the global sizing problem): the reliability method gives a probability with an error of 10^{-2} compared to Monte-Carlo (figure 8). This error is truly negligible for our case of interest and allows us to believe that the reliability method can be systematically used for robust design.

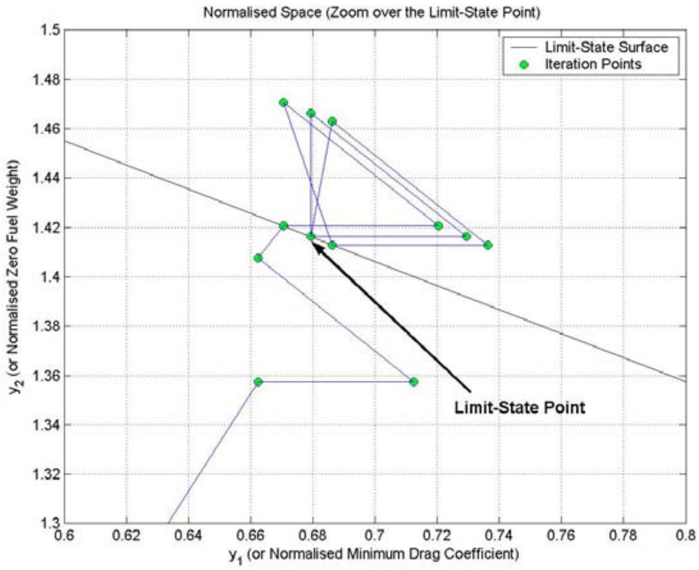

Figure 7: Iterative process of constrained optimization for FORM method.

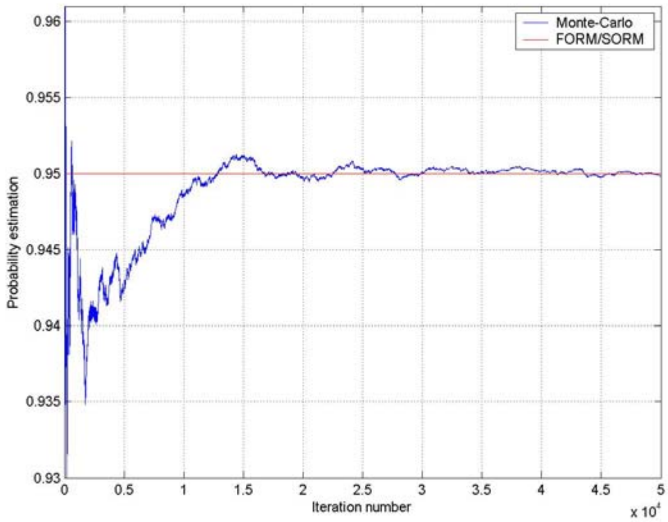

Figure 8: Comparison between probabilities evaluated with Monte-Carlo and FORM methods.

3.0 DESIGN OF A ROBUST AIRCRAFT

Using the general framework to manage uncertainties, the preliminary design process described in paragraph 1.0 can be adapted such that the decision criteria, $P(R < 4000 \text{ NM}) = 0.05$, is satisfied.

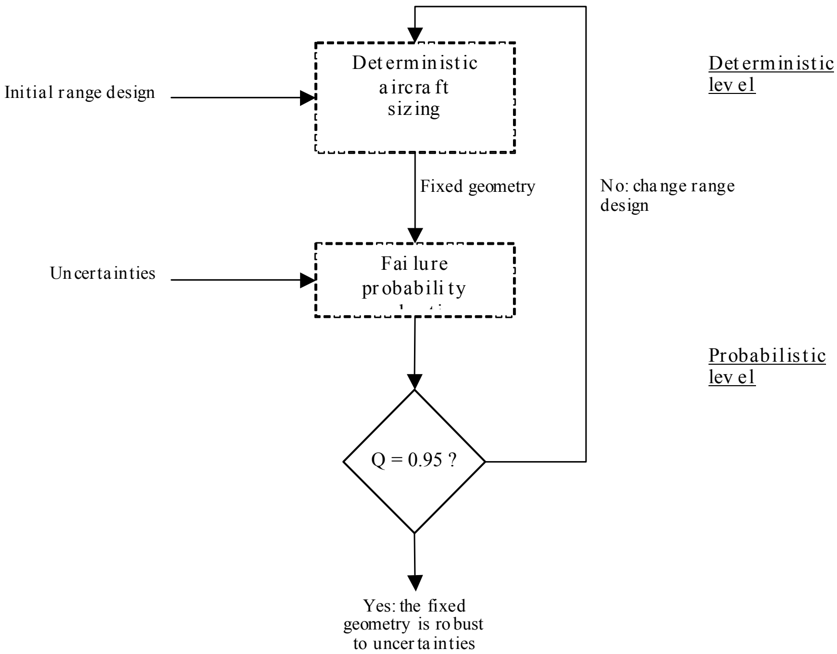

Figure 9: Robust design process of an aircraft.

The iterative process is composed of two following steps: design loop using a objective Range Design, computation of the probability $P(R < 4000 \text{ NM})$, and modification of the Range Design consign in order to reduce the so-called failure probability (figure 9).

In this robust design process, each uncertainty propagation method leads to a robust aircraft. The characteristics of the designed aircrafts are given in table 2.

Characteristics	Moments method	Monte-Carlo method	Relative difference
Body length [m]	40.2	40.4	-0.5%
Fuel Mass [kg]	28991	29578	-2%
Empty weight [kg]	23394	23598	-0.9%
Take-off weight [kg]	53493	54186	-1.3%
Mean range [NM]	4230	4280	-1.2%
Standard deviation of R [NM]	120	130	-7.7%
Probability of specified performance	0.95	0.95	

Table 2: Different uncertainty propagation methods give different aircraft (the probability of success is computed by the propagation method).

The difference between the two “robust aircraft” does not seem to be very significant, but a deviation of 500 kg on an aircraft has a real economic impact on its success.

This work on the global management of uncertainties is being complemented by a study of uncertainties related to Computational Fluid Dynamics (CFD) computations. This is carried out in the European project NODESIM-CFD[1], starting in November 2006. We will concentrate our efforts on moment methods.

[1] NODESIM-CFD is a collaborative research project funded by the EC under the 6th PCRD (contract # 030959)

4.0 USE OF CFD RESULTS

Some uncertain variables involved in the aircraft performance computation are outputs of high fidelity models. For instance, the drag coefficient is computed by a CFD code. Then, uncertainty assessment in this numerical simulation will provide important information on quantification of uncertainties. This R&D axis will improve the robust design process sketched in the previous paragraph.

The first-order second moment (FOSM) method, very popular in uncertainty analysis, uses a linearization of the function that relates the input variables and parameters to the output variables. This approximation occasionally leads to problems when the mean value of the input variables is close to a local/global extremal value of the function. In this case, the FOSM computes artificially a zero uncertainty because the first derivative of the function is close to zero. To overcome this, we have to use a quadratic reconstruction, instead of a linear one: second derivatives of the observation function are required.

In CFD this can be done using a differentiation of the adjoint equations. Using available automatic differentiation tools, these derivatives can be obtained by differentiating the code computing the adjoint, which is already the differential in tangent mode of the state equation. In NODESIM-CFD, Dassault-Aviation is investigating this approach for a 2D Reynolds- Averaged Navier-Stokes (RANS) solver.

It is also interesting to notice that the quadratic approximation can also be used as a fast surrogate model to perform Monte-Carlo simulation, thus enabling comparisons between the studied moment methods and a Monte-Carlo method.

5.0 FORTHCOMING WORK

The next step towards the elaboration of a complete robust design process of an aircraft will be the extension of the methodology to more precise and complex physical models. In the context of the multilevel modelling, the next challenges will be:

- The management of uncertainties through the different levels of modelling;
- To overcome the technological obstacle of uncertainty propagation through complex system, such that CPU costly physical models.

Since the distribution of uncertain variables should have an influence on satisfaction of the decision criteria, these two goals will provide a more precise quantification of uncertainties in the context of global design. This step is necessary before developing a robust multi-physical optimization process which constitutes the future big issue in complex systems design.

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